Closing *Tues*: 10.2
Closing Fri: 3.5(1)(2)

3.5 Implicit Differentiation (*continued*)

Given any equation of the form:

$$F(x, y) = 0,$$

we think of *y* as an *implicit* function of *x*

$$F(x, y(x)) = 0$$

and differentiate directly (**correctly using the chain rule as we go**) to find dy/dx.

Entry Task: Find the equation for the tangent line to

$$y^2 = x$$

at $(x, y) = (4, -2)$.

Find dy/dx.

1. $x^4 y + y^3 = x$

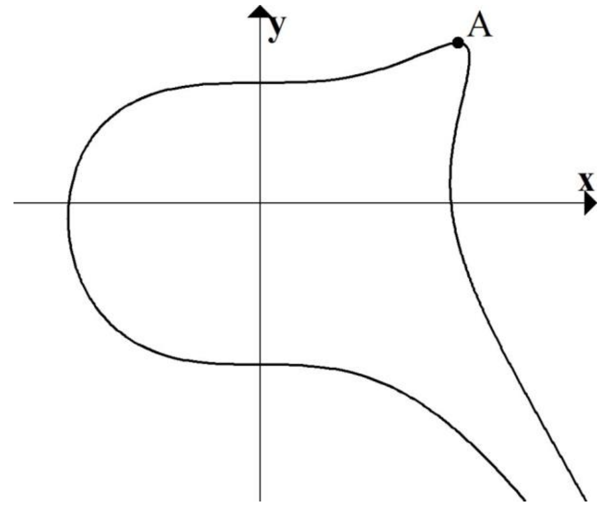
2. $xe^y + \tan(x) + \sin(y) = 1$

Old Midterm Question:

Consider the curve implicitly defined by

$$(x^3 - y^2)^2 + e^y = 4.$$

Find the (x, y) coordinates of the point *A* shown (highest point on the curve).



Inverse Functions: We write inverse functions as $y = f^{-1}(x)$ which is equivalent to $f(y) = x$.

We can implicitly differentiate

$$\frac{d}{dx}[f(y) = x] \Rightarrow f'(y)\frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{f'(y)}$$

Examples: Find dy/dx

1. $y = \sqrt{x}$

2. $y = \sin^{-1}(x)$

3. $y = \tan^{-1}(x)$

$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$	$\frac{d}{dx}(\cot^{-1}(x)) = -\frac{1}{1+x^2}$
$\frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{\mathrm{x}\sqrt{x^2-1}}$	$\frac{d}{dx}(\csc^{-1}(x)) = -\frac{1}{\mathrm{x}\sqrt{x^2-1}}$

- *Note*: The formulas all assume the principal domains as defined in our textbook.

Now you can use these *shortcuts*.

Exercise: Find dy/dx

$$y = \tan^{-1}(e^{3x})$$

3.6 Derivatives of Logarithms and Logarithmic Differentiation

<u>Quick test of basic understanding</u>

Solve $3^x + 1 = 11$

Recall your logarithm facts:

1. $y = \ln(x) \quad \leftrightarrow \quad e^y = x$

2. $e^{\ln(x)} = x \quad$ and $\quad \ln(e^y) = y$

3. $\ln(ab) = \ln(a) + \ln(b)$
   $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$
   $\ln(x^n) = \text{n} \ln(x)$

4. $y = \log_a(x) \leftrightarrow \quad a^y = x$
   (so $\ln(x) = \log_e(x)$)

Find the derivative of $y = \ln(x)$

Find the derivative of $y = \log_{\mathrm{a}}(x)$

Power functions:

$$\frac{d}{dx}\left[\left(g(x)\right)^n\right] = n\left(g(x)\right)^{n-1} g'(x)$$

Example:

$$\frac{d}{dx}[(x^3 + 2x)^{10}] =$$

Exponential functions:

$$\frac{d}{dx}\left[e^{g(x)}\right] = e^{g(x)} g'(x)$$

$$\frac{d}{dx}\left[a^{g(x)}\right] = a^{g(x)} \ln(a) g'(x)$$

Examples:

$$\frac{d}{dx}\left[e^{(x^4-5x)}\right] =$$

$$\frac{d}{dx}\left[7^{(x^4-5x)}\right] =$$

What if the variable *x* is in BOTH the base and exponent?

Example: $y = (3x + 1)^x$

Answer: *Logarithmic Differentiation*

Step 1: Take log of both sides

Step 2: Differentiate implicitly

Step 3: Solve for y’.